

حل تمارين الأعداد والحساب

تمارين (الكتاب المدرسي) الجزء الأول

أصحح أم خاطئ؟



(1) $\frac{1}{7}$ ينتمي إلى: $\mathbb{Q}$ و $\mathbb{I}$.

(2) العدد الطبيعي هو: $\frac{\sqrt{52}}{\sqrt{13}}$ لأن $\frac{\sqrt{52}}{\sqrt{13}} = \sqrt{\frac{52}{13}} = \sqrt{4} = 2$.

(3) العدد غير العشري هو: $\frac{1}{3 \times 10^2}$ لأنه لا يمكننا كتابة مقامه بالشكل: 10^n .

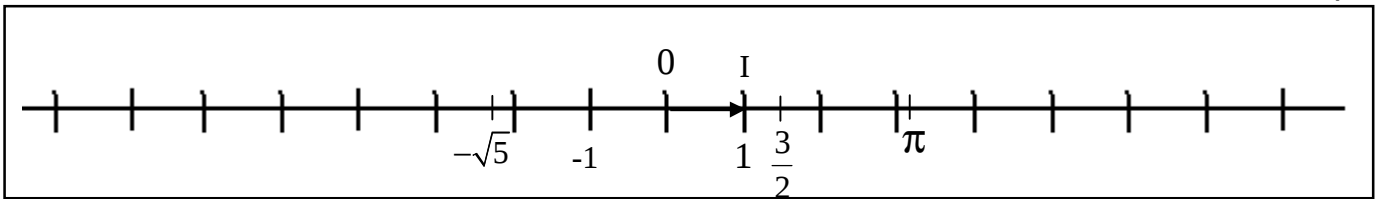
(4) العدد الأولي هو: **121** لأنه يقبل القسمة على 1 وعلى نفسه فقط، $37 \times 7 = 259$ و $61 \times 3 = 183$.

(5) التحليل المناسب للعدد 6270 هو: **$2 \times 3 \times 5 \times 209$** .

(6) العدد **$225 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$** .

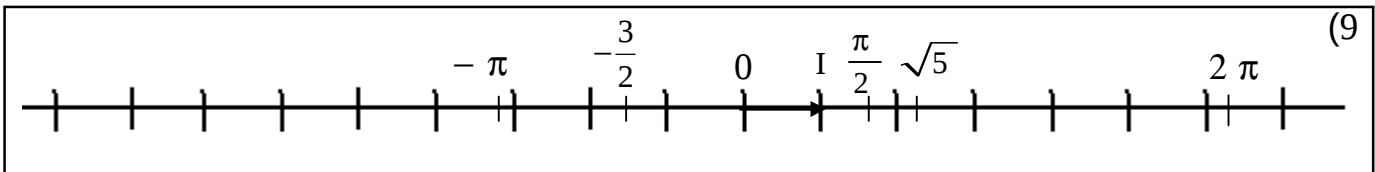
تمثيل أعداد على المستقيم العددي:

(7)

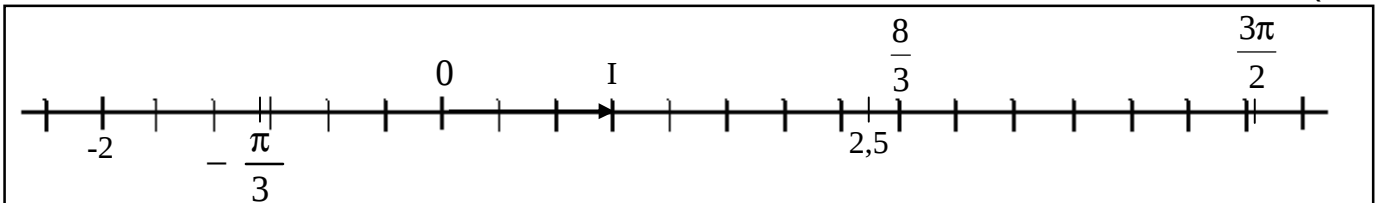


(8) $x = 5,17$; $y = 5,25$; $z = 5,36$.

(9)



(10)



مجموعات الأعداد:

(11) $\frac{2\pi}{3} \in \mathbb{I}$; $\frac{\sqrt{2}}{3} \notin \mathbb{Q}$; $\frac{1}{3} \notin \mathbb{D}$; $3,5 \notin \mathbb{C}$; $10 \in \mathbb{Z}$

(12)



الأعداد	¥	Z	D	□	i
-3		×	×	×	×
125	×	×	×	×	×
$2\sqrt{3}$					×
$\frac{1}{\sqrt{2}}$					×
$-\frac{7}{3}$				×	×
π					×
0	×	×	×	×	×
2,75			×	×	×

(13)

$$\bullet A = \frac{-\sqrt{144}}{3} = \frac{-12}{3} = -4 \Leftrightarrow A \in \mathbf{Z}.$$

$$\bullet B = \frac{\pi}{3,14} = \frac{3,14}{3,14} = 1 \Leftrightarrow B \in \mathbf{¥}.$$

$$\bullet C = \sqrt{2} - \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}(\sqrt{2}-1)-1}{\sqrt{2}-1} = \frac{2-\sqrt{2}-1}{\sqrt{2}-1} = \frac{1-\sqrt{2}}{\sqrt{2}-1} = \frac{-(\sqrt{2}-1)}{\sqrt{2}-1} = -1 \Leftrightarrow C \in \mathbf{Z}.$$

$$I = [-4; 3] \quad (14)$$

- $I_{¥} = \{0; 1; 2; 3\} \Leftrightarrow$ عدد العناصر هو 4.
- $I_{\mathbf{Z}} = \{-4; -3; -2; -1; 0; 1; 2; 3\} \Leftrightarrow$ عدد العناصر هو 8.
- عدد عناصر $\square$ التي تشملها I هو عدد غير محدود.

(15)

الأعداد	¥	Z	D	□	i
58	×	×	×	×	×
$\frac{3}{2}$			×	×	×
$-\frac{15}{3}$		×	×	×	×
$1,5 \times 10^3$	×	×	×	×	×
2π					×
$\frac{1}{100}$			×	×	×
$\sqrt{64}$	×	×	×	×	×
$(0,5)^2$			×	×	×

(16)

الأعداد	3,503	10^{-3}	-10^3	3587	4×10^{-2}	π	$-\frac{3}{100}$	$-\frac{22}{7}$	3,14
الأعداد العشرية	×	×	×	×	×		×		×
الأعداد الناطقة غير العشرية								×	
الأعداد غير الناطقة						×			
الأعداد	$\sqrt{2}$	$\sqrt{0,25}$	$\frac{1}{3}$	$-\frac{21}{6}$	$\frac{2}{\pi}$	$\sqrt{\pi}$	$\sqrt{\sqrt{16}}$	0	
الأعداد العشرية		×		×			×	×	
الأعداد الناطقة غير العشرية			×						
الأعداد غير الناطقة	×				×	×			

(17) تذكير: العدد الناطق هو الذي يكتب على شكل كسر $\frac{p}{q}$ غير قابل للاختزال، أي أن p و q أوليان فيما

بينهما، والعدد العشري هو الذي يكتب على الشكل $\frac{a}{10^n}$ أو $\frac{a}{2^m \times 5^n}$ ، ولدينا أيضا $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{D} \subset \mathbb{R}$



• إذن 2,5 هو عدد ناطق. $2,5 = \frac{25}{10} = \frac{5}{2}$

• -0,47 هو عدد عشري إذن فهو عدد ناطق. $-0,47 = -\frac{47}{10^2}$

• 120 عدد طبيعي إذن فهو عدد ناطق.

• عدد عشري إذن فهو عدد ناطق. $\frac{0,125}{62,5} = \frac{125}{62500} = \frac{125}{625 \times 10^2} = \frac{1}{5 \times 10^2} = \frac{1}{2^2 \times 5^3}$

• عدد عشري إذن فهو عدد ناطق. $\frac{5}{40 \times 10^{-2}} = \frac{5}{40 \times 10^{-2}} = \frac{5}{0,4} = 12,5$

(18)

$25000 \times 10^{-4} = 2,5$
$52 \times 10^{-3} = 0,052$
$3 \times 10^{-2} = 0,03$
$12 \times 10^6 = 12000000$
$6,125 \times 10^{-3} = 0,006125$
$62,39 \times 10^4 = 623900$

(19)

• $A = 0,027027.....$

نفرض $x = 0,027027$ لدينا إذن: $1000x = 27,027.....$

$1000x = 27 + 0,027.....$

$1000x = 27 + x$

$$999x = 27$$

$$x = \frac{27}{999} = \frac{1}{37}$$

$$A = \frac{1}{37} \quad \text{ومن هنا فإن}$$

$$B = 34,1456\ 456\ldots \bullet$$

$$(1) \ 10B = 341,456\ 456 = 341 + 0,456\ 456\ldots$$

$$1000x = 456,456\ldots \quad \text{لنفرض } x = 0,456\ 456\ldots \text{ لدينا إذن:}$$

$$1000x = 456 + 0,456\ldots$$

$$1000x = 456 + x$$

$$999x = 456$$

$$(2) \ x = \frac{456}{999} = \frac{152}{333}$$

$$10B = 341 + x = 341 + \frac{152}{333} \quad \text{من (1) و (2) لدينا:}$$

$$10B = \frac{341 \times 333 + 152}{333} = \frac{113705}{333}$$

$$B = \frac{113705}{333 \times 10} = \frac{113705}{3330}$$

$$B = \frac{113705}{3330} \quad \text{إذن}$$

(20) تذكير: العدد العشري هو الذي يكتب على الشكل $\frac{a}{10^n}$ أو $\frac{a}{2^m \times 5^n}$ ، ولدينا أيضا $\mathbb{Y} \subset \mathbb{Z} \subset \mathbb{D}$:

الأعداد	$\frac{1}{2000}$	$\frac{33}{375}$	$-\frac{32}{105}$	$\frac{71}{25}$	$\frac{15}{4}$	$-\frac{13}{12}$	$\frac{3}{2}$
كتابة العدد بطريقة أخرى	$\frac{1}{2^4 \times 5^3}$	$\frac{11}{5^3}$	$-\frac{2^5}{3 \times 5 \times 7}$	$\frac{71}{5^2}$	$\frac{15}{2^2}$	$-\frac{13}{2^2 \times 3}$	
الأعداد العشرية	×	×		×	×		×
الأعداد الناطقة			×			×	

(21) باستعمال الحاسبة ودون لمسة الفاصلة نكتب كما يلي:

$$38 \times 10^{-5} \times 329562 \times 10^{-4} = 12523356 \times 10^{-9} = 0,012523356$$

(22)

$\frac{1}{100} = 10^{-2}$
$\left(\frac{1}{10}\right)^2 = \frac{1^2}{10^2} = 10^{-2}$
$(2 \times 50)^{-2} = (10^2)^{-2} = 10^{-4}$

ومنه فإن: $\frac{1}{100} = \left(\frac{1}{10}\right)^2 = 10^{-2}$

(23)

$\frac{0,21}{1,05} = \frac{21}{105} = \frac{3 \times 7}{3 \times 5 \times 7} = \frac{1}{5} \Leftrightarrow \frac{0,21}{1,05} \in \mathbf{D}$
$\frac{7\pi+14}{3\pi+6} = \frac{7(\pi+2)}{3(\pi+2)} = \frac{7}{3} \Leftrightarrow \frac{7\pi+14}{3\pi+6} \in \mathbf{\pi}$
$\frac{16}{6} - \frac{11}{3} = \frac{8}{3} - \frac{11}{3} = -\frac{3}{3} = -1 \Leftrightarrow \frac{16}{6} - \frac{11}{3} \in \mathbf{Z}$
$-\frac{6\pi}{3} = -2\pi \Leftrightarrow -\frac{6\pi}{3} \in \mathbf{i}$
$\frac{2}{\sqrt{2}+1} - 2\sqrt{2} = \frac{2 - 2\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}+1}$ $= \frac{2 - 2 \times 2 - 2\sqrt{2}}{\sqrt{2}+1} = \frac{-2(\sqrt{2}+1)}{\sqrt{2}+1} = -2$ $\Leftrightarrow \frac{2}{\sqrt{2}+1} - 2\sqrt{2} \in \mathbf{Z}$



(24)

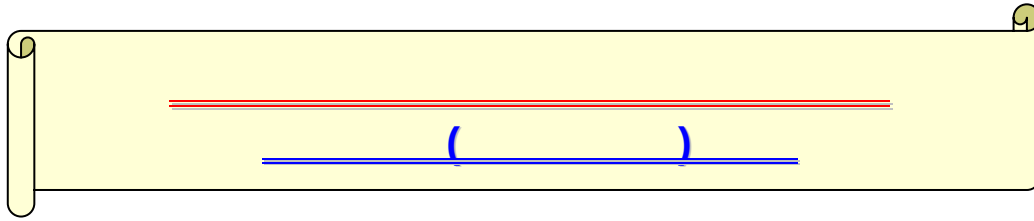
$$A = (589 - 32) \div (633 + 917) = 0,359..... \Leftrightarrow A = \frac{589 - 32}{633 + 917} \quad \bullet$$

$$B = 7 \div (2 \times 3 \times 4) = 0,2916666.... \Leftrightarrow B = \frac{7}{2 \times 3 \times 4} \quad \bullet$$

$$C = \sqrt{(56,25 + 7,75)} - 8 = 0 \Leftrightarrow C = \sqrt{56,25 + 7,75} - 8 \quad \bullet$$

- $3a + \frac{1}{a} + 2 \Leftrightarrow 3 \times a + (1 \div a) + 2$
- $\frac{3a + 1}{a + 2} \Leftrightarrow (3 \times a + 1) \div (a + 2)$
- $\frac{3(a + 1)}{a} + 2 \Leftrightarrow (3 \times (a + 1) \div a) + 2$
- $3a + \frac{1}{a + 2} \Leftrightarrow 3 \times a + 1 \div (a + 2)$
- $\frac{3(a + 1)}{a + 2} \Leftrightarrow [3 \times (a + 1)] \div (a + 2)$





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(26)

$$\cdot \quad 5 \quad (-3)^5 < 0 \quad \bullet$$

$$\cdot \quad 8 \quad (-5)^8 > 0 \quad \bullet$$

$$\cdot \quad -243 = -3^5 \quad -3^5 < 0 \quad \bullet$$

$$\cdot \quad 0,001 = 10^{-3} \quad 10^{-3} > 0 \quad \bullet$$

$$\cdot \quad 2 \quad (-3^3)^2 > 0 \quad \bullet$$

(27)

- $2^2 \times 3^3 = 4 \times 27 = 108$.
- $2^2 \times 3^3 \times 5 = 4 \times 27 \times 5 = 108 \times 5 = 540$.
- $2^3 \times 3^3 = 8 \times 27 = 216$.
- $2^3 \times 3^2 = 8 \times 9 = 72$.
- $\left(\frac{1}{2}\right)^2 \times 3^3 = \frac{1}{4} \times 27 = \frac{27}{4}$.
- $\left(\frac{1}{3}\right)^2 \times \left(\frac{3}{2}\right)^3 = \frac{1}{3^2} \times \frac{3^3}{2^3} = \frac{3}{8}$.
- $2 \times \left(\frac{1}{2}\right) \times \left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$.
- $\left(-\frac{1}{2}\right)^2 \times \left(-\frac{1}{3}\right)^3 = \frac{1}{2^2} \times \left(-\frac{3^3}{2^3}\right) = -\frac{27}{32}$.
- $\left(-\frac{5}{4}\right) \times \left(\frac{4}{5}\right)^2 \times \left(-\frac{2}{5}\right) = \frac{(-5) \times 4^2 \times (-2)}{4 \times 5^2 \times 5} = \frac{8}{25}$.



(28)

$$A = \frac{(-2)^5 \times (-6)^3 \times (-3)^8}{(15)^2 \times (-12)^3}$$

$$A = \frac{(-2)^5 \times (-2)^3 \times 3^3 \times (-3)^8}{3^2 \times 5^2 \times (-4)^3 \times 3^3} = \frac{2^8 \times 3^8}{3^2 \times 5^2 \times (-1)^3 \times 2^6}$$

$$A = \frac{2^2 \times 3^6}{-25} = -\frac{4 \times 729}{25} = -\frac{2916}{25}$$

(29)

$$\bullet A = (2^3 \times 2^{-4})^2 \times (3^3)^2 \times 3^{-5}$$

$$A = (2^3 \times 2^{-4})^2 \times (3^3)^2 \times 3^{-5}$$

$$A = (2^{-1})^2 \times 3^6 \times 3^{-5} = 2^{-2} \times 3 = \frac{3}{4}$$

$$\bullet B = 2^3 \times 2^4 \times 2^{-5} = 2^{3+4-5} = 2^2 = 4.$$

$$\bullet C = \left(\frac{2}{3}\right)^2 \times 3^3 = \frac{2^2 \times 3^3}{3^2} = 4 \times 3 = 12.$$

$$\bullet D = (2^3 \times 3^2)^2 = 2^6 \times 3^4 = 64 \times 81 = 5184.$$

$$\bullet E = \left(-\frac{1}{3}\right)^2 \times 5^{-2} \times \left(\frac{3}{5}\right)^3 = \frac{1}{3^2} \times \frac{1}{5^2} \times \frac{3^3}{5^3} = \frac{3}{5^5} = \frac{3}{3125}.$$

$$\bullet F = \left(\frac{2}{7}\right)^4 \times \left(\frac{7}{4}\right)^2 \times \left(\frac{-49}{2}\right)^3 = \frac{2^4}{7^4} \times \frac{7^2}{2^4} \times \left(-\frac{7^2}{2}\right)^3 = \frac{1}{7^2} \times (-1) \times \frac{7^6}{2^3} = -\frac{7^4}{8} = -\frac{2401}{8}$$

$$\bullet G = \left(\frac{2}{3}\right)^{-2} \times \left(\frac{3}{4}\right)^4 \times \left(\frac{27}{4}\right)^{-1} = \frac{3^2}{2^2} \times \frac{3^4}{4^4} \times \frac{4}{27} = \frac{3^6}{4^4 \times 3^3} = \frac{3^3}{4^4} = \frac{27}{256}.$$

(30)

$$\bullet a = \frac{2^4}{10^5} = \frac{2^4}{2^5 \times 5^5} = 2^{-1} \times 5^{-5}.$$

$$\bullet b = \frac{25^3}{5^{-5}} = \frac{5^6}{5^{-5}} = 5 = 2^0 \times 5^1.$$

$$\bullet c = \frac{(10^2)^3}{2^6 \times 5^6} = \frac{10^6}{2^6 \times 5^6} = \frac{2^6 \times 5^6}{2^6 \times 5^6} = 1 = 2^0 \times 5^0.$$

(31)

$$\bullet A = \frac{12^5 \times 35^{-2}}{49^{-3} \times 21^4} = \frac{3^5 \times 4^5 \times 5^{-2} \times 7^{-2}}{(7^2)^{-3} \times 3^4 \times 7^4}$$

$$A = \frac{3 \times 4^5 \times 5^{-2} \times 7^{-2}}{7^{-6} \times 7^4} = \frac{3 \times 4^5}{5^2} = \frac{3072}{25}$$

$$\bullet B = \frac{(-5)^3 \times (-8)^3 \times (-9)^2}{15^2 \times 12^2} = \frac{(2^3)^3 \times 5^3 \times (3^2)^2}{3^4 \times 5^2 \times 4^2}$$

$$B = \frac{2^9 \times 5 \times 3^4}{3^4 \times 2^4} = 2^5 \times 5 = 160$$



(32

$$A = 987891236^2 - 987891235^2$$

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$$A = 987891236^2 - 987891235^2 = 1976000000$$

.

$$6 \neq 5$$

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$$a = 987891236$$

•

$$A = a^2 - (a - 1)^2 = a^2 - (a^2 + 1 - 2a) = 2a - 1$$

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$$A = 2 \times 987891236 - 1 = 1975782471$$

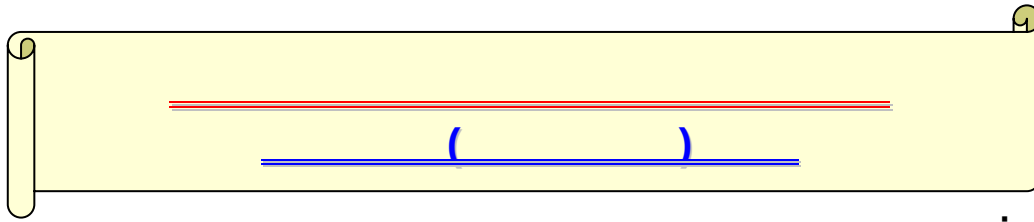
$$987891235^2 \quad 987891236^2$$

•

A

.1976000000





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(33)

$\sqrt{\pi} = 1,772453..... \Leftrightarrow \sqrt{\pi} \in \mathbb{R}$
$\sqrt{(-3)^2} = \sqrt{3^2} = 3 \Leftrightarrow \sqrt{(-3)^2} \in \mathbb{R}$
$\sqrt{-25} \notin \mathbb{R}$
$\sqrt{13} - \sqrt{136} = \sqrt{13 - 11,66...} = 1,3380.... \Leftrightarrow \sqrt{13 - \sqrt{136}} \in \mathbb{R}$
$\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3} \Leftrightarrow \sqrt{\frac{5}{9}} \in \mathbb{R}$

(34)



$\sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$
$\sqrt{6} \times \sqrt{48} = \sqrt{6 \times 48} = \sqrt{6 \times 6 \times 2^2 \times 2} = 12\sqrt{2}$
$\sqrt{\frac{75}{27}} = \sqrt{\frac{25 \times 3}{9 \times 3}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$
$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

(35)

$A = 2\sqrt{3} + 5\sqrt{12} - \sqrt{75} = 2\sqrt{3} + 10\sqrt{3} - 5\sqrt{3} = 7\sqrt{3}$
$B = 3\sqrt{80} - \sqrt{180} - \sqrt{45} = 12\sqrt{5} - 6\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$

(36)

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$
$\frac{2}{\sqrt{2}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = \sqrt{2}$
$\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$$\cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \sqrt{8} - \sqrt{2} = \frac{2}{\sqrt{2}} = \sqrt{2} :$$

(37)

$$E = \left(\frac{12 + 25\sqrt{6}}{6} \right) \div \left(1 + \frac{\sqrt{6}}{4} - \frac{1}{\sqrt{6}} + \sqrt{24} \right)$$

$$E = \left(\frac{12 + 25\sqrt{6}}{6} \right) \div \left(1 + \frac{\sqrt{6}}{4} - \frac{\sqrt{6}}{6} + 2\sqrt{6} \right) = \left(\frac{12 + 25\sqrt{6}}{6} \right) \div \left(\frac{12 + 3\sqrt{6} - 2\sqrt{6} + 24\sqrt{6}}{12} \right)$$

$$E = \left(\frac{12 + 25\sqrt{6}}{6} \right) \div \left(\frac{12 + 25\sqrt{6}}{12} \right) = \left(\frac{12 + 25\sqrt{6}}{6} \right) \times \left(\frac{12}{12 + 25\sqrt{6}} \right) \Leftrightarrow E = 2$$



$$: \frac{1 + \sqrt{5}}{2} \quad (38)$$

$$\frac{1}{1 + \sqrt{5}} = \frac{2}{1 + \sqrt{5}} = \frac{2(1 - \sqrt{5})}{(1 + \sqrt{5})(1 - \sqrt{5})} = \frac{2(1 - \sqrt{5})}{1 - 5} = \frac{2(1 - \sqrt{5})}{-4} = -\frac{1 - \sqrt{5}}{2} = \frac{\sqrt{5} - 1}{2}.$$

(39)

$\sqrt{81} = 9$
$\sqrt{175} = \sqrt{25 \times 7} = 5\sqrt{7}$
$\sqrt{1080} = \sqrt{36 \times 30} = 6\sqrt{30}$
$\sqrt{27} + \sqrt{48} = 3\sqrt{3} + 4\sqrt{3} = 7\sqrt{3}$
$\sqrt{0,45} = \sqrt{\frac{45}{100}} = \frac{3\sqrt{5}}{10}$
$\sqrt{27} + 2\sqrt{75} - \sqrt{108} = 3\sqrt{3} + 10\sqrt{3} - 6\sqrt{3} = 7\sqrt{3}$
$\sqrt{36} - 3\sqrt{6} + 5\sqrt{144} = 6 - 3\sqrt{6} + 5 \times 12 = 6 + 60 - 3\sqrt{6} = 66 - 3\sqrt{6}$
$\sqrt{\frac{8}{9}} \times \sqrt{\frac{12}{25}} \times \sqrt{\frac{225}{24}} = \frac{2\sqrt{2}}{3} \times \frac{2\sqrt{3}}{5} \times \frac{15}{2\sqrt{6}} = \frac{2 \times 2 \times 15\sqrt{6}}{2 \times 15\sqrt{6}} = 2$

(40)

$(1 + \sqrt{2})^2 = 1 + 2 + 2\sqrt{2} = 3 + 2\sqrt{2}$
$(2\sqrt{5} + 3)^2 = 4 \times 5 + 9 + 12\sqrt{5} = 29 + 12\sqrt{5}$
$(1 - 5\sqrt{2})^2 = 1 + 25 \times 2 - 10\sqrt{2} = 51 - 10\sqrt{2}$
$(7 - \sqrt{3})(7 + \sqrt{3}) = 7^2 - (\sqrt{3})^2 = 49 - 3 = 46$
$(1 - \sqrt{2})(1 + \sqrt{3}) \times 2\sqrt{2} = (1 + \sqrt{3} - \sqrt{2} - \sqrt{6}) \times 2\sqrt{2}$ $= 2\sqrt{2} + 2\sqrt{6} - 2 \times 2 - 2\sqrt{12}$ $= 2\sqrt{2} - 4\sqrt{3} + 2\sqrt{6} - 4$

(41)

$\frac{5}{2\sqrt{10}} = \frac{5\sqrt{10}}{20} = \frac{\sqrt{10}}{4}$
$\frac{\sqrt{7}}{3\sqrt{5}} = \frac{\sqrt{35}}{3 \times 5} = \frac{\sqrt{35}}{15}$
$\frac{\sqrt{2}+1}{\sqrt{2}+2} = \frac{(\sqrt{2}+1)(\sqrt{2}-2)}{(\sqrt{2}+2)(\sqrt{2}-2)} = \frac{2-2\sqrt{2}+\sqrt{2}-2}{2-4} = \frac{-\sqrt{2}}{-2} = \frac{\sqrt{2}}{2}$
$\frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{4+3-4\sqrt{3}}{4-3} = \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$

(42)



$A = \frac{2}{\sqrt{33}} \left(\frac{\sqrt{363}}{\sqrt{2}-1} \right) = \frac{2}{\sqrt{33}} \left(\frac{11\sqrt{3}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \right)$ $A = \frac{2}{\sqrt{33}} \left(\frac{11\sqrt{6}+11\sqrt{3}}{2-1} \right) = \frac{22\sqrt{6}+22\sqrt{3}}{\sqrt{33}}$ $A = \frac{(22\sqrt{6}+22\sqrt{3}) \times \sqrt{33}}{33} = \frac{22\sqrt{198}+22\sqrt{99}}{33}$ $A = \frac{2 \times 11 \times 3\sqrt{22} + 2 \times 11 \times 3\sqrt{11}}{33}$ $A = 2\sqrt{22} + 2\sqrt{11}$
$B = \frac{3\sqrt{360} - 2\sqrt{180}}{\sqrt{10} - \sqrt{2}} = \frac{3 \times 6\sqrt{10} - 2 \times 6\sqrt{5}}{\sqrt{10} - \sqrt{2}}$ $B = \frac{(18\sqrt{10} - 12\sqrt{5})(\sqrt{10} + \sqrt{2})}{(\sqrt{10} - \sqrt{2})(\sqrt{10} + \sqrt{2})}$ $B = \frac{18 \times 10 + 18 \times 2\sqrt{5} - 12 \times 5\sqrt{2} - 12\sqrt{10}}{10 - 2}$ $B = \frac{180 + 36\sqrt{5} - 60\sqrt{2} - 12\sqrt{10}}{8}$ $B = \frac{45 + 9\sqrt{5} - 15\sqrt{2} - 3\sqrt{10}}{2}$

$$(1) \quad a + b = 1 \quad a^2 + b^2 = 2 \quad (43)$$

$$\bullet \quad (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Leftrightarrow 2ab = (a + b)^2 - (a^2 + b^2) = 1 - 2 = -1 \Leftrightarrow ab = -\frac{1}{2} = -0,5.$$

$$\bullet \quad a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = (a^2 + b^2)^2 - 2(ab)^2$$

$$= 2^2 - 2\left(-\frac{1}{2}\right)^2 = 4 - 2 \times \frac{1}{4} = 4 - \frac{1}{2} = \frac{8-1}{2} = \frac{7}{2} = 3,5.$$

$$\bullet \quad \begin{aligned} & a^4 + b^4 \\ & b = \frac{1+\sqrt{3}}{2} \quad a = \frac{1-\sqrt{3}}{2} \end{aligned}$$

$$\bullet \quad a + b = \frac{1-\sqrt{3}}{2} + \frac{1+\sqrt{3}}{2} = \frac{1-\sqrt{3}+1+\sqrt{3}}{2} = \frac{2}{2} = 1.$$

$$\bullet \quad a^2 + b^2 = \left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2$$

$$= \frac{1+3-2\sqrt{3}}{4} + \frac{1+3+2\sqrt{3}}{4}$$

$$= \frac{4-2\sqrt{3}+4+2\sqrt{3}}{4} = \frac{8}{4} = 2$$



(1)

(44)

$$\bullet \quad \sqrt{2} \times \sqrt{2+\sqrt{3}} = \sqrt{2 \times (2+\sqrt{3})} = \sqrt{4+2\sqrt{3}} \quad (1)$$

$$\bullet \quad (1+\sqrt{3})^2 = 1+3+2\sqrt{3} = 4+2\sqrt{3} \Leftrightarrow 1+\sqrt{3} = \sqrt{4+2\sqrt{3}} \quad (2)$$

$$1+\sqrt{3} = \sqrt{4+2\sqrt{3}} = \sqrt{2} \times \sqrt{2+\sqrt{3}} \quad (2) \quad (1)$$

$$\bullet \quad \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}}$$

$$= \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} + \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{2-\sqrt{3}}{4-3} = \sqrt{2}-1 + \sqrt{3}-\sqrt{2} + 2-\sqrt{3}$$

$$= \sqrt{2}-\sqrt{2} + \sqrt{3}-\sqrt{3} -1 + 2 = 1$$

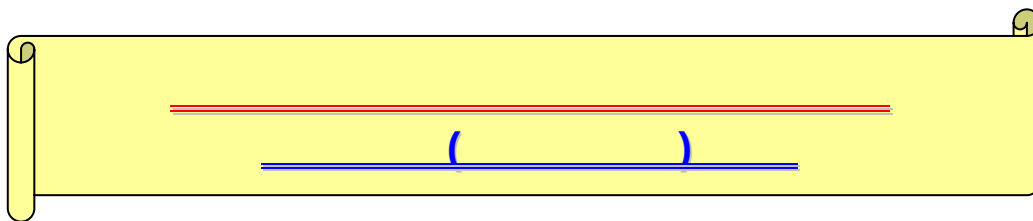
$$E = x^2 - 3x + 4 \quad (45)$$

$$: \quad x = 1 + \sqrt{3}$$

$$E = (1 + \sqrt{3})^2 - 3(1 + \sqrt{3}) + 4$$

$$= 1 + 3 + 2\sqrt{3} - 3 - 3\sqrt{3} + 4$$

$$= 5 - \sqrt{3}$$



:

(46)

$\frac{2000}{7} = 285,714$
$\frac{\pi}{60} = 0,052$
$\cos 80^\circ = 0,174$
$\frac{3\sqrt{7} - 9}{2} = -0,531$



(47)

$1205\sqrt{3} \times 4.10^{-4}$	$\frac{3\pi}{2}$	
0,835	4,712	10^{-3}

(48)

$-34,56 \times 10^{-2}$	$5,03 \times 10^{-4}$	$6,5 \times 10^5$	12×10^{-3}	
$-3,456 \times 10^{-1}$	/	/	$1,2 \times 10^{-2}$	

(49)

150×10^{-3}	$27,31 \times 10^3$	0,095	251,3	
$1,50 \times 10^{-1}$	$2,731 \times 10^4$	$9,5 \times 10^{-2}$	$2,513 \times 10^2$	
2×10^{-1}	3×10^4	1×10^{-1}	3×10^2	

(50)

$$A = 9 \times 10^{-3} + 0,4 \times 10^{-2} - 9 \times 10^{-4}$$

$$A = 9 \times 10^{-3} + 4 \times 10^{-3} - 0,9 \times 10^{-3}$$

$$A = (9 + 4 - 0,9) \times 10^{-3}$$

$$A = 12,1 \times 10^{-3}$$

$$A = 1,21 \times 10^{-2}$$

$$:149 \times 10^6 \text{ km} \quad (51)$$

$$149 \times 10^6 \text{ km} = 149 \times 10^6 \times 10^3 \text{ m} = 149 \times 10^9 \text{ m}.$$

$$v = \frac{d}{t} \Leftrightarrow t = \frac{d}{v} = \frac{149 \times 10^9}{3 \times 10^8} = 49,66 \times 10 = \mathbf{496,6s}.$$

(52)

$\frac{181,47}{78,956}$	$0,05 \times 1200 \times 10^{-3}$	$851,7 \times 0,0018 \times 0,073$	
2,298	0,06	0,1119	
3×10^0	6×10^{-2}	1×10^{-1}	

(53)

$$.8,5 \times 10^{19} = \frac{8,96 \times 10^{-6}}{1,05 \times 10^{-25}} = \quad \div \quad = \quad 1\text{mm}^3$$

(54)

$(ab)^2$	ab	a^2b^2	b^2	a^2	b	a
2×10^{-11}	4×10^{-6}	2×10^{-11}	4×10^{-29}	5×10^{17}	6×10^{-15}	7×10^8

 $\cdot (ab)^2$ a^2b^2

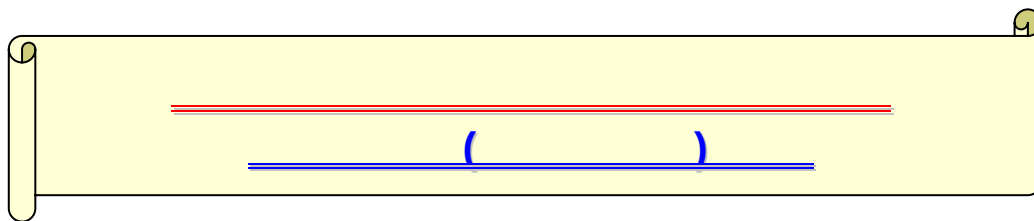
(55)

$$R = 6371 \text{ km} = 6,371 \times 10^7 \text{ cm}.$$

$$v = \frac{4\pi \times R^3}{3} = \frac{4 \times \pi \times (6,371 \times 10^7)^3}{3} = 1,08 \times 10^{24} \text{ cm}^3.$$

$$\mu = \frac{m}{v} \Leftrightarrow m = \mu \times v = 5,5 \times 1,08 \times 10^{24} = 5,94 \times 10^{24} \text{ g} = \mathbf{5,94 \times 10^{18} \text{ T}}.$$





:

(56)

49 27 21 .3

9

. ... 81

1

2

(57)



$3^3 = 27$ 3	27	197
	23	101
$3 \times 6 = 18$ 3	18	89
$3^4 \times 5 = 405$ 5 3	405	43
$11 \times 29 = 319$ 29 11		319

$$.7 \times 37 = 259 \quad 7 \quad 37$$

(58) 259

(59)

$$.2^3 \times 3^2 \times 5 \times 7 = 2520$$

$$.7951 \times 1 = 7951$$

(60)

$$.7+5=12$$

$$5.+5=10$$

$$5.+3=8$$

$$.3+3=6$$

$$.2+2=4$$

$$.13+7=20$$

$$.11+7=18$$

$$11.+5=16$$

$$7.+7=14$$

:

$$.n \in \mathbb{N} \quad P(n) = n^2 + n - 41 \quad (61)$$

4	3	2	1	0	n
-21	-29	-35	-39	-41	P(n)

$$15 \quad P(n) = 15 \quad n = 7$$

$$1 + 2 + 4 + 7 + 14 = 28 \quad 28 : 30 \quad 25 \quad (62)$$

(63)

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$$

 $2n+1$

$$13 = 12 + 1 = 2 \times 6 + 1 \Leftrightarrow 13 = (6+1)^2 - 6^2 = 7^2 - 6^2 = 49 - 36 = 13.$$

$$45 = 44 + 1 = 2 \times 22 + 1 \Leftrightarrow 45 = (22+1)^2 - 22^2 = 23^2 - 22^2 = 529 - 484 = 45.$$

(64)

$$S = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}$$

$$S = \frac{30}{2 \times 30} + \frac{10}{6 \times 10} + \frac{5}{12 \times 5} + \frac{3}{20 \times 3} + \frac{2}{30 \times 2}$$

$$S = \frac{30+10+5+3+2}{60} = \frac{50}{60} = \frac{5}{6}$$

$$A = \frac{1}{n} - \frac{1}{n+1}$$

$$A = \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n^2+n}$$

$$\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n^2+n}$$



$$S = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}$$

$$S = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6}$$

$$S = \frac{1}{1} - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

5	4	3	2	1	n
$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{2}$	A

(65)

$$A = \frac{(-4)^2(-25)^3}{36 \times 10^2} = \frac{(4)^2(-1)^3(25)^3}{6^2 \times (2 \times 5)^2} = \frac{2^4(-1)^3(5^2)^3}{(2 \times 3)^2 \times 2^2 \times 5^2} = \frac{-2^4 \times 5^6}{2^4 \times 3^2 \times 5^2} = \frac{-5^4}{3^2} = \frac{-625}{9}$$

$$B = \frac{6\sqrt{288} \times \sqrt{75}}{\sqrt{90} \times \sqrt{20}} = \frac{2 \times 3\sqrt{2^5 \times 3^2} \times \sqrt{3 \times 5^2}}{\sqrt{9 \times 10} \times \sqrt{2 \times 10}}$$

$$B = \frac{2^3 \times 3^2 \sqrt{2} \times 5\sqrt{3}}{3\sqrt{10} \times \sqrt{2} \times \sqrt{10}} = \frac{2^3 \times 3 \times 5\sqrt{3}}{10} = \frac{2^3 \times 3 \times 5\sqrt{3}}{2 \times 5} = 12\sqrt{3}$$

(66)

- $\frac{48}{75} = \frac{2^4 \times 3}{3 \times 5^2} = \frac{16}{25}$.
- $\frac{180}{126} = \frac{2^2 \times 3^2 \times 5}{2 \times 3^2 \times 7} = \frac{10}{7}$.
- $\frac{585}{1275} = \frac{3^2 \times 5 \times 13}{3 \times 5^2 \times 17} = \frac{3 \times 13}{5 \times 17} = \frac{39}{85}$.
- $\frac{17303}{792} = \frac{11^3 \times 13}{2^3 \times 3^2 \times 11} = \frac{11^2 \times 13}{2^3 \times 3^2} = \frac{1573}{72}$.

 $a\sqrt{b}$

74

 $\sqrt{74}$: _____ (67)
. $\sqrt{84}$

- $\sqrt{54} = \sqrt{2 \times 3^3} = 3\sqrt{2 \times 3} = 3\sqrt{6}$.
- $\sqrt{84} = \sqrt{2^2 \times 3 \times 7} = 2\sqrt{3 \times 7} = 2\sqrt{21}$.
- $\sqrt{845} = \sqrt{5 \times 13^2} = 13\sqrt{5}$.
- $\sqrt{1000} = \sqrt{2^3 \times 5^3} = 2 \times 5\sqrt{2 \times 3} = 10\sqrt{10}$.
- $\sqrt{20825} = \sqrt{17 \times 35^2} = 35\sqrt{17}$.

(68)

- $330 = 2 \times 3 \times 5 \times 11$.
- $252 = 2^2 \times 3^2 \times 7$.
- $\frac{330}{252} = \frac{2 \times 3 \times 5 \times 11}{2^2 \times 3^2 \times 7} = \frac{5 \times 11}{2 \times 3 \times 7} = \frac{55}{42}$.
- $\sqrt{252} = \sqrt{2^2 \times 3^2 \times 7} = 2 \times 3\sqrt{7} = 6\sqrt{7}$.



A

24

 $A = 2^3 \times 5^2 \times 7$: _____ (69)
.28

- $A = 2^3 \times 5^2 \times 7 = 2^2 \times 2 \times 5^2 \times 7 = 2^2 \times 7 \times 2 \times 5^2 = 28 \times 50$

.A 28

r q, p $2^p \times 5^q \times 7^r$

kA

- $A = 2^3 \times 5^2 \times 7 \Leftrightarrow kA = 2^4 \times 5^2 \times 7^2 \Leftrightarrow k = \frac{2^4 \times 5^2 \times 7^2}{2^3 \times 5^2 \times 7} = 2 \times 7 = 14$.

.14 k

$$kA = 2^4 \times 5^2 \times 7^2 = 19600 \Leftrightarrow \sqrt{kA} = \sqrt{19600} = 140 : \underline{\hspace{2cm}}$$

$$r \quad q, p \quad 2^p \times 5^q \times 7^r \quad mA \quad \cdot$$

$$\cdot \quad A = 2^3 \times 5^2 \times 7 \Leftrightarrow mA = 2^3 \times 5^3 \times 7^3 \Leftrightarrow m = \frac{2^3 \times 5^3 \times 7^3}{2^3 \times 5^2 \times 7} = 5 \times 7^2 = 245.$$

$$mA = 2^3 \times 5^3 \times 7^3 = 343000 \Leftrightarrow \sqrt[3]{mA} = \sqrt[3]{343000} = 70 : \underline{\hspace{2cm}}$$

$$f_n = 2^{2^n} + 1 \quad (70)$$

3	2	1	0	n
257	17	5	3	f_n

$$.641 \quad \cdot \quad f_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297 = 641 \times 6700417 \quad \cdot$$

$$n \quad 2^n - 1 \quad (71)$$

7	5	3	2	1	0	n
127	31	7	3	1	0	$2^n - 1$

$$n \quad 11 \quad \cdot \quad 2^{11} - 1 = 2047 = 23 \times 89 \quad \cdot$$

(72)

- $\text{pgcd}(45;105) = 15.$
- $\frac{45}{105} = \frac{15 \times 3}{15 \times 7} = \frac{3}{7}$
- $\sqrt{45} = \sqrt{3^2 \times 5} = 3\sqrt{5}.$
- $45 \times 105 = 3^2 \times 5 \times 3 \times 5 \times 7 = 3^3 \times 5^2 \times 7.$
- $45^4 = (3^2 \times 5)^4 = 3^8 \times 5^4.$
- $105^3 = (3 \times 5 \times 7)^3 = 3^3 \times 5^3 \times 7^3.$



(73)

:pgcd(420;378)

$$\text{pgcd}(420;378) = 42$$

$$378 \div 42 = 9$$

$$420 \div 42 = 10$$

. 9

10

(74)

$$\blacksquare A^2 = 4^3 \times 15^4 \times 11^2 = (2^2)^3 \times (3 \times 5)^4 \times 11^2 = 2^6 \times 3^4 \times 5^4 \times 11^2.$$

$$\blacksquare A^2 = 2^6 \times 3^4 \times 5^4 \times 11^2 \Leftrightarrow A = \sqrt{2^6 \times 3^4 \times 5^4 \times 11^2} = \sqrt{(2^3 \times 3^2 \times 5^2 \times 11)^2} = 2^3 \times 3^2 \times 5^2 \times 11$$

(75)

: 240

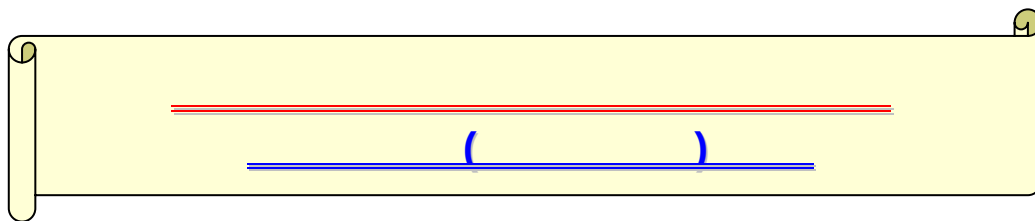
240×n

n

$$: n = 3 \times 5 = 15 \quad 240 = 2^4 \times 3 \times 5$$

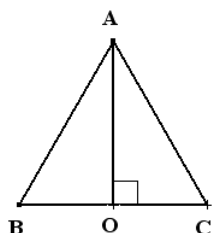
$$. 240 \times n = 2^4 \times 3 \times 5 \times 3 \times 5 = 2^4 \times 3^2 \times 5^2 = (2^2 \times 3 \times 5)^2$$





:

$$909\ 526 \quad p = 2^{3021377} - 1 \quad p \quad (76)$$



:

O

.2 ABC (77

.[BC]

[AO]

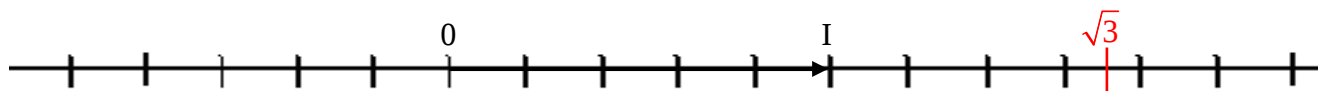
[AO]

ABC

AOC

$$AO^2 = AC^2 - OC^2 = 2^2 - 1^2 = 4 - 1 = 3$$

$$AO^2 = 3 \Leftrightarrow AO = \sqrt{3}$$



$$39 = 3 \times 13$$

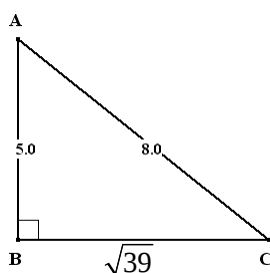
$$39 = (a - b) \times (a + b) \left\{ \begin{array}{l} a - b = 3 \\ a + b = 13 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} a = 8 \\ b = 5 \end{array} \right.$$

$$39 = (8 - 5) \times (8 + 5) = 8^2 - 5^2 \Leftrightarrow \sqrt{39} = \sqrt{8^2 - 5^2}$$

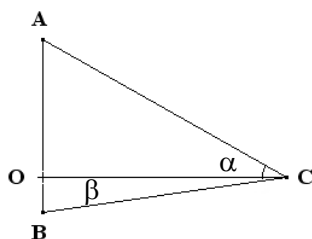
8cm

 $\sqrt{39}$. $\sqrt{39}$

5cm



(78



- $\tan \alpha = \frac{AO}{OC} \Leftrightarrow AO = \tan \alpha \times OC = \tan 30^\circ \times 64,3 = 0,577 \times 64,3 = 37,1 \text{ m}.$
- $\tan \beta = \frac{OB}{OC} \Leftrightarrow OB = \tan \beta \times OC = \tan 2,45^\circ \times 64,3 = 0,042 \times 64,3 = 2,7 \text{ m}.$
- $AC = AO + OB = 37,1 + 2,7 = 39,8 \approx 40 \text{ m}.$

.40 m

(79

.	L	:	R1	▪
.L-1	1	:	R2	▪
$\boxed{\frac{L}{1} = \frac{1}{L-1}} : \quad R1 \quad R2 \quad \cdot$				

- $c = \frac{L}{1} \Leftrightarrow L = c \times 1$

$$\begin{aligned} \frac{L}{1} = \frac{1}{L-1} &\Leftrightarrow \frac{c \times 1}{1} = \frac{1}{c \times 1 - 1} \\ &\Leftrightarrow (c \times 1)(c \times 1 - 1) = 1^2 \\ &\Leftrightarrow c^2 \times 1^2 - c \times 1^2 = 1^2 \\ &\Leftrightarrow 1^2(c^2 - c) = 1^2 \\ &\Leftrightarrow c^2 - c = \frac{1^2}{1^2} \\ &\Leftrightarrow c^2 - c = 1 \Leftrightarrow c^2 - c - 1 = 0 \end{aligned}$$

- $\left(c - \frac{1}{2}\right)^2 = \frac{5}{4} \Leftrightarrow c^2 + \frac{1}{4} - 2 \times \frac{1}{2} \times c = \frac{5}{4} \Leftrightarrow c^2 + \frac{1}{4} - c - \frac{5}{4} = 0 \Leftrightarrow c^2 - c - 1 = 0 \Leftrightarrow 0 = 0$

- $\left(c - \frac{1}{2}\right)^2 = \frac{5}{4} \Leftrightarrow \left(c - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2 = 0 \Leftrightarrow \left[\left(c - \frac{1}{2}\right) - \frac{\sqrt{5}}{2}\right] \left[\left(c - \frac{1}{2}\right) + \frac{\sqrt{5}}{2}\right] = 0$

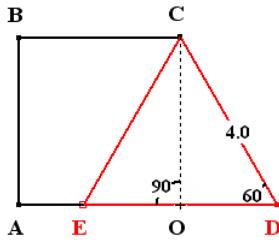
$$\Leftrightarrow \begin{cases} \left(c - \frac{1}{2}\right) - \frac{\sqrt{5}}{2} = 0 \\ \left(c - \frac{1}{2}\right) + \frac{\sqrt{5}}{2} = 0 \end{cases} \Leftrightarrow \begin{cases} c - \frac{1}{2} = \frac{\sqrt{5}}{2} \\ c - \frac{1}{2} = -\frac{\sqrt{5}}{2} \end{cases} \Leftrightarrow \begin{cases} c = \frac{1}{2} + \frac{\sqrt{5}}{2} \\ c = \frac{1}{2} - \frac{\sqrt{5}}{2} \end{cases} \Leftrightarrow \begin{cases} c = \frac{1+\sqrt{5}}{2} \\ c = \frac{1-\sqrt{5}}{2} \end{cases}$$

$$\frac{1+\sqrt{5}}{2} > 0$$

$$.c = \frac{1+\sqrt{5}}{2} :$$

$$. \frac{1-\sqrt{5}}{2} < 0 \Leftrightarrow 1-\sqrt{5} < 0 \Leftrightarrow 1 < \sqrt{5} :$$





$$\hat{D} = 60^\circ \quad CD = 4\text{cm}$$

$$CD = DE = EC = 4\text{cm}$$

$$[ED] \quad [ED]$$

ABCD (80

CDE

O

▪

▪

:

[CO]

CDE

▪

:

O

COD

$$EO = OD = 2\text{cm}$$

$$[ED] \quad [CO]$$

▪

$$CO = 3,4 \text{ cm} \quad CO^2 = CD^2 - OD^2 = 4^2 - 2^2 = 16 - 4 = 12$$

$$CD^2 = CO^2 + OD^2$$

$$BC = AO \quad AB = CO = 3,4\text{cm} :$$

ABCO

$$\hat{A} = \hat{B} = \hat{O} = 90^\circ$$

▪

$$P1 = CD + DE + EC = 12\text{cm}$$

CDE

P1

▪

.ABCE

P2

▪

$$P2 = AB + BC + CE + AE$$

$$P2 = AB + AO + CE + AE$$

$$P2 = 3,4 + AE + 2 + 4 + AE$$

$$P2 = 2AE + 9,4$$



$$P1 = P2 \Leftrightarrow P2 = 12 \Leftrightarrow 2AE + 9,4 = 12 \Leftrightarrow 2AE = 12 - 9,4 = 2,6 \Leftrightarrow AE = 1,3\text{cm}.$$

$$BC = a; AC = b; AB = c$$

.A

ABC (81

$$S1 = \frac{bc}{2} : ABC$$

S1

▪

$$S2 = \frac{\pi \times \left(\frac{c}{2}\right)^2}{2} = \frac{\pi \times \frac{c^2}{4}}{2} = \frac{\pi \times c^2}{8} : [AB]$$

S2

▪

$$S3 = \frac{\pi \times \left(\frac{a}{2}\right)^2}{2} = \frac{\pi \times \frac{a^2}{4}}{2} = \frac{\pi \times a^2}{8} : [BC]$$

S3

▪

$$S4 = \frac{\pi \times \left(\frac{b}{2}\right)^2}{2} = \frac{\pi \times \frac{b^2}{4}}{2} = \frac{\pi \times b^2}{8} : [AC]$$

S4

▪

:

S

▪

$$S = S1 + S2 + S4 - S3 = \frac{bc}{2} + \frac{\pi \times c^2}{8} + \frac{\pi \times b^2}{8} - \frac{\pi \times a^2}{8} = \frac{4bc + \pi(c^2 + b^2 - a^2)}{8}.$$

:

A

ABC

▪

$$BC^2 = AB^2 + AC^2 \Leftrightarrow a^2 = c^2 + b^2$$

$$S = \frac{4bc + \pi(a^2 - a^2)}{8} = \frac{4bc}{8} = \frac{bc}{2}.$$

.ABC

▪